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Rest masses of elementary particles as basic information of Gibbs–Falkian thermodynamics

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Dedicated to Karl Stephan, Stuttgart, on the occasion of his seventieth birthday

Abstract — As developed by G. Falk (following J.W. Gibbs), methods of modern thermodynamics allow by means of the Einstein mechanics to describe as well energy–momentum transports (EMT) which occur under vacuum conditions. Such EMT *relations* are proved to manifest a set of elementary particles which in its entirety determines real state changes of fluids on the macro-level. EMT differ from each other by their masses at rest ($m_{\#}$) resulting as integration constant each. In order to identify $m_{\#}$, Seelig has established a new theory whose physical background is de Broglie's mechanics of matter waves. Seelig's results stand out without exception by their excellent agreement with a lot of measured $m_{\#}$ values of elementary particles. Strikingly, the structure of the Seelig equation even permits for an extension upon particles hitherto outside Seelig's approach. This new semi-empirical mass formula agrees with the experimental $m_{\#}$ values of *all* known stabile elementary particles better than 0.085 % on weighted average. The formula has some outstanding properties leading to some remarkable conclusions on Higgs particles and allowing predictions of particles hitherto unknown. Thus, a new key to a uniform understanding of real processes in physics, e.g., in thermal sciences, is now at hand: matter is realized on the micro-level by a finite number of particle classes which are subject to well-posed constraints holding on macro-level for the physical system in question. © 2000 Éditions scientifiques et médicales Elsevier SAS

thermodynamics / Gibbs–Falkian dynamics / Einstein mechanics / energy–momentum transports / elementary particles / mass at rest / Seelig equation / semi-empirical mass formula / Higgs particles / thermal sciences

Nomenclature

μ^{t}	chemical potential (per particle)	J
ρ^{m}	density (molar)	mol·m^{-3}
∅	diameter (particle average)	nm
$E_{\#}$	energy (at state of rest)	kg·m^{2}·s^{-2}, eV
S	entropy	J·K^{-1}
p, q, v, w	exponents in SEMF	integer number
$\boldsymbol{F}$	force vector	kg·m·s^{-2}
ν	frequency	s^{-1}, Hz
$m_{\#}$	mass (at rest per particle)	eV·c^{-2}
m^{t}	mass (permanent per particle)	eV·c^{-2}
$\boldsymbol{P}$	momentum (linear)	kg·m·s^{-1}
$\boldsymbol{i}$	momentum (specific)	m·s^{-1}
N	number of a certain kind of particles	
$\boldsymbol{r}$	position vector	m
p	pressure	kg·m^{-1}·s^{-2}, bar
T	temperature	K, °C
V	volume	m^{3}
V_{P}^{msp}	volume (hypothetical, molar)	m^{3}·mol^{-1}
λ	wavelength	m, nm

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"We want to use pure logic as far as we can and then calculate away from the resulting theory. And find: my God, it agrees with the things we observe."

(Brian Greene on String Theory, original quotation from Die Zeit, Issue 7, 10 February 2000, p. 43)

1. SOME CONSIDERATIONS OF ELEMENTARY PARTICLES

1.1. Matter concepts today

Strangely enough the ancient ideas of atoms still occur in today's particle models, although certain characteristic

TABLE I
Parameters [1].

Quantity	Symbol, equation	Value
Avogadro constant	N_A	$6.0221367\cdot10^{23}$ mol^{-1}
Bohr radius	$a_\infty = 4\pi\varepsilon_0\hbar^2/m_e e^2 = r_e\alpha^{-2}$	$0.529177249\cdot10^{-10}$ m
classical electron radius	$r_e = e^2/4\pi\varepsilon_0 m_e c^2$	$2.81794092\cdot10^{-15}$ m
dilatation factor	$f_k = \lvert 1 - 1/(3\pi)\rvert^{-1/2}$	1.057684967...
electron charge magnitude	e	$1.60217733\cdot10^{-19}$ C
electron mass	m_e	0.51099906 MeV$\cdot c^{-2}$
fine-structure constant	$\alpha = e^2/4\pi\varepsilon_0\hbar c$	1/137.0359895
molar volume, ideal gas at STP	V_A	$22.41410\cdot10^{-3}$ m$^3\cdot$mol^{-1}
permittivity of free space	ε_0	$8.854187817...\cdot10^{-12}$ F$\cdot$m^{-1}
Planck constant	h	$6.6260755\cdot10^{-34}$ J$\cdot$s
proton mass	m_p	938.27231 MeV$\cdot c^{-2}$
Rydberg energy	$hcR_\infty = m_e c^2\alpha^2/2$	13.60569814 eV
Seelig factor	$\varsigma = [(3\pi)^3/(2\pi)] \cdot \lvert 1 - 1/(3\pi)\rvert^{-1/4}$	137.0287449...
speed of light in vacuum	c	299 792 458 m$\cdot$s^{-1}

restrictions are imposed. They are used in various kinetic or statistical theories to symbolize corpuscles in *billiard ball* arrangements or mathematical mass points, respectively. The so tagged “particle” has no spatial extension, but can be provided with a mass value, and perhaps an electrical charge. The modern concept of matter is ambiguous in comparison to Democritus’ atomism. He had in mind the permanence of atoms. Metaphorically speaking: one changes one’s appearance and moods while retaining one’s identity. Hence, continued identity is permanence. In other words: the property of things is given by their having a substance or — equivalent to this statement — by their being fully permanent. Today even this permanence is denied. However, modern theories confirm the ancient idea that matter is composed of many particles — constituents — which evolve from the microscopic scale towards the continuously larger distances on the macroscopic level. Common matter is formed from molecules, atoms, and (free) electrons. The molecules comprise atoms, they in turn consist of (bound) electrons and a nucleus, which includes protons and neutrons, and these again are constituted of quarks and gluons. [1] With reference to the atomic level governed by orthodox quantum theory or to the sub-atomic level of elementary particles, there is no evidence that the so defined constituents be indivisible or immortal in the strict sense of the ancient philosophy of nature. And they are, furthermore, assumed to take up no space, i.e. they are Eulerian mass-points (cf. [2, p. 138]). Nevertheless, these two antique principles, demanding indivisibility and a timeless existence, could survive due to the high degree of abstraction of the theory. *Significant is now the unchangeable “law” with respect to transformations in space, time, and many other coordinates*: on the subatomic level the characteristic rules of symmetry constitute the laws of nature for the elementary particles. Their interactions are primarily dependent on the gravitational and electromagnetic forces, as well as the so-called weak and strong nuclear ones.

A crucial point of current matter concepts concerns the usual notion of a vacuum. It lost its traditional identity with a general void or an empty space believed to be the platform for microscopic events. Nowadays, on the atomic level the (*Dirac*) *vacuum* represents a definite state of minimum energy which is consistent with the boundary conditions prescribed for the system in question. This ground state has a zero point energy giving rise to vacuum fluctuations connected with real or virtual vacuum polarizations.

On a subatomic level the vacuum structure is entirely different. At present, there is a particular interest in the

[1] Recently physicists have discovered a fresh source of many Nobel prices for the next generation. A team of physicists at the famous Fermi lab has found by experiments under 400 billions electron volts that quarks are probably extended and might be divided. The theorists call these constituents “preons” or “haplons”. If a substructure below the quark level were confirmed all current theories (e.g., superstrings, big bang, etc.) would have to be revised.

so-called gluon vacuum. Gluons are the gauge particles of the strong interactions between the quarks. The vacuum constituted by gluons is defined to be "invisible", indicating the fact that these elementary particles do very efficiently influence themselves mutually (they become "lumpy"). But neither do gluons interact with electrons or photons nor with nucleons, though these protons or neutrons consist of quarks which for their part cannot individually exist within the electrically neutral (Dirac) vacuum. A nucleus is considered to be an ensemble composed of nucleon bubbles moving unhindered inside the lumpy gluon vacuum.

1.2. Role of the Einstein mechanics for real fluids

Since 1905 precisely, the evolution of modern science and technology was gravely influenced by Albert Einstein's *Special Theory of Relativity* (STR). The central part of the STR is, however, of much more general relevance than its results concerning in particular relativistic motions. It identifies *Newton–Eulerian mass-point mechanics* as a special case of extended mechanics, called *Einstein mechanics*.

Objectives of the new mechanics are focussed on an adequate quantitative representation of elementary particles as definite *energy–momentum transports*. The crucial point concerns the problem to understand real fluids like a gas, liquid, or solid under prescribed state conditions as a restricted ensemble of an enormous number of diverse, but structured energy–momentum transports.

Einstein's key to establish a new mechanical method of matter representation consists in abandoning one of the axioms which constitute the Newton–Eulerian mass-point mechanics. The equivalence relation between the (linear) *momentum* $\boldsymbol{P}$ of a particle and Newton's *quantity of motion* (*quantitas motus*) $m^{\mathrm{t}}\boldsymbol{v}$ postulates the identity of the *local particle flow velocity* $\boldsymbol{v}$ with the *specific momentum* $\boldsymbol{i}$, i.e. $\boldsymbol{v} \equiv \boldsymbol{i} = \boldsymbol{P}/m^{\mathrm{t}}$, where m^{t} means the permanent mass per particle as a characteristic property of matter.

Starting point of Einstein mechanics is the opposite idea which may be clarified by the apparently simple mathematical relationship

$$\boldsymbol{P} = m(E)\boldsymbol{v} \tag{1.1}$$

where the *mass* $m(E)$ is now a function of the (total) energy assigned to a number N of a certain kind of particles like electrons or neutrons. Clearly, this number of particles forms a system establishing its motion by the velocity $\boldsymbol{v}$

$$\boldsymbol{v} = \frac{\partial E(\boldsymbol{P}, \boldsymbol{r}, V, S, N)}{\partial \boldsymbol{P}} \tag{1.2}$$

The term *system* is used according to the Gibbs–Falkian dynamics [3]. The latter is in no way confined to physics (cf. [4]). Hence, to avoid misunderstandings about our contribution to physics, we prefer the notion *Gibbs–Falkian* **Thermo***dynamics* in the title of our paper at least.

Section 2 contains the details of Einstein mechanics applied to a definite process under hard *vacuum conditions*. The latter allow to reduce equation (1.1) to the form

$$\boldsymbol{P} = m\big(E(\boldsymbol{P})\big)\boldsymbol{v} \tag{1.3}$$

The involved dependence of the energy E on the momentum $\boldsymbol{P}$ is fixed by a function $E(\boldsymbol{P}, E_{\#})$ denoted as *energy–momentum transport*. Falk and Ruppel evidenced $E(\boldsymbol{P}, E_{\#})$ as a synonym of the corresponding particle, provided the respective process is subject to hard vacuum and the *homogenous* function $E(\boldsymbol{P}, E_{\#})$ is related to a single particle (cf. [5, pp. 49 f.]).

An option to distinguish the different kinds of particles from each other is available by the *rest energy* $E_{\#}$ defined by reference to the limiting case of vanishing momentum $\boldsymbol{P}$

$$\lim E(\boldsymbol{P}, E_{\#}) \to E_{\#} \quad \text{for } \boldsymbol{P} \to \boldsymbol{0} \tag{1.4}$$

Referred to a single particle the *rest mass* $m_{\#}$ *per particle* results from the equivalence relation

$$\frac{E_{\#}}{N} \equiv m_{\#}c^2 \tag{1.5}$$

by definition and serves to catalogue the different kinds of particles. Hence, the relevance of the parameter $m_{\#}$ for physics and technology is evident.

Note an electrically neutral gas under common standard conditions. Then, the corresponding values of the state quantities *pressure* p (760 Torr), *temperature* T (0 °C) and *mole density* ρ^{m} (44.647 mol·m^{-3}) refer to a definite number of particles denoted as *Avogadro number* N_{A}. The decisive point is that this amount manifests an important conservation law concerning the so-called *baryon–lepton constancy* [6, pp. 80 f.]. This means for the common physico–chemical level at pressure values between some Torrs and, say, 1 000 bars or values of temperature up to some 1 000 K that *chemical reactions* occur, where the baryons (protons etc.) and leptons (electrons etc.) involved do not change their total number. Only their rearrangements arise due to exchange processes between the diverse molecules and atoms or certain fractions of such particles. Clearly, the result of such rearrangements depends on the prescribed values of pressure and temperature in accordance with the laws proved to be true for chemical reactions. On the micro-level corresponding scattering processes happen mediated in particular by photons for which no conservation law holds. All these events induce the respective particles to shape certain dynamic structures tending towards states of equilibrium. For instance, some characteristic averaged spatial distances appear between the *heavy* particles involved (molecules, atoms, ions, protons, neutrons).

In order to describe those particles by means of a unified representation like the concept of *energy–momentum transports*, some physical requirements need to be fulfilled. Certainly, the most important condition concerns the available space within the volume, where the transports take place. Section 2 deals with the constraints to be imposed on any ensemble of $E(\boldsymbol{P}, E_{\#})$ transports. Above all, sufficient empty space is mandatory, expressed by a certain value of the ratio that may be defined by the *mole volume* V_{A} (22.414 $\mathrm{dm^3{\cdot}mol^{-1}}$) at standard conditions related to the hypothetical volume $V_{\mathrm{P}}^{\mathrm{msp}}$ of all particles. The latter is obtained by summing up the spherical volumes of all N_{A} particles, provided they can be arranged as closed-packing structure. Assuming an averaged particle diameter σ of about $\sigma \approx 0.3$ nm (He: $\sigma \approx 0.2$ nm; CO_2: $\sigma \approx 0.4$ nm) a rough approximation yields first the value $V_{\mathrm{Co}} \approx 241\ \mathrm{cm^3{\cdot}mol^{-1}}$ for the **C**o-volume of all N_{A} particles, then the value $V_{\mathrm{P}}^{\mathrm{msp}} \approx 169\ \mathrm{cm^3{\cdot}mol^{-1}}$, and, finally, about 100 for the ratio $V_{\mathrm{A}}/V_{\mathrm{P}}^{\mathrm{msp}}$. In other words, under standard conditions the spatial extensions of all particles take up an amount of 1 % compared with the spatial extension of V_{A}. Thus, the hypothesis seems conclusive that the Einstein mechanics is a powerful tool to understand the micro-level behavior of matter as a special kind of exchange processes between diverse ensembles of *energy–momentum transports*. As to its dynamics each $E(\boldsymbol{P}, E_{\#})$ transport can be represented in a unified manner and identified by its rest energy $E_{\#}$. The latter allows to be reduced to a parameter $m_{\#}$ that is usually interpreted as the rest mass per particle of the ensemble under consideration. (An extension of the Einstein mechanics for more pressurized matter was published in [7, p. 89].)

1.3. State of the art

As to particle physics the state of things in general is still dominated by the *Standard Model* (SM). The latter is a special application of renormalizable quantum field theory at low energies and allows one to derive some quantitative predictions for experimental results whose success confirms the validity of that theory. However, the assumption of the SM — especially, the *axiom of renormalizability* along with the *gauge symmetry principles* — rules out certain events such as the decay of isolated protons and forbids the *neutrinos* from having masses. Unlike this prediction, there is, indeed, some evidence for upper limits of an *electron-neutrino mass*, which were recently established (cf. [8–10]).

The hitherto proved elementary particles exhibit a wide range of masses that disobey any recognizable pattern, with the electron 350 000 times lighter than the heaviest quark, and neutrinos even lighter.

The SM works with a mechanism which accounts for any of these masses in such a way that the interactions of the so-called *scalar fields* with fields like the electric and magnetic ones are assumed to give the respective particles their masses. In other words, the task to confirm the existence of these scalar fields is equivalent to discover new elementary particles.

But all these interactions depend on the energy at which they are measured. This experience is the key of understanding particle physics in so far as the realm of, e.g., the SM is bounded by energies which produce some pretty big mass ratios like the cited 350 000 to 1 ratio of the top quark to the electron mass. It is striking that this is nothing compared with the enormous ratio of the fundamental *unification energy scale* of about 10^{16} GeV (or 10^{18} GeV in case *gravitational forces* are included) to the energy scale of about 100 GeV which is typical of the SM. This huge scale results from the extrapolation of the interactions of the SM fields into the region, where they all become equal to one another at an energy of a little more than 10^{16} GeV.

Special problems arise in connection with *quark masses* and their values tabulated in the data listings. Any serious information on a quark mass value is related to an exclusive computational method to be used for extracting the mass values from observations. Due to different methods being in use, the quark mass values in the particle data listings can be significantly different [11]. In summary, if using the data listings, it should be emphasized that the numerical value for a quark mass is irrelevant without further informations on the exclusive method by which it was obtained (cf. [11]).

Some years ago evidence has appeared that the string theories together with a competed quantum field theory in 11 dimensions are all special versions of a universal basic theory (sometimes denoted *M-Theory*) under different premises about reality and its representation. It is striking that at present no one really knows how to formulate the M-Theory mathematically. By far the greatest obstacle to achieve this objective is the fact that nobody has the faintest idea about the physical principles governing the basic theory (cf. [12, 13]). However, it should be stressed that each version of those string theories succeeds in dealing with an aspect of enormous relevance: for a long time, it seemed impossible to develop a modification of quantum theory based on anything but *point* particles. The truly impressive feature of string theory is that about twenty years of intensive research have demonstrated how to avoid this basic assumption of quantum mechanics in its common practice. Indeed, the extended nature of the *one-dimensional* string prevents the immediate consequence of any *zero-dimensional* point-particle concept, namely, the pernicious infinities responsible for the central dilemma of contemporary theoretical physics. But might it also be the case that an infinitely thin one-dimensional strand is similarly a mathematical idealization? Might it actually be the case that strings have some thickness? Such higher-dimensional basic objects actually do play an important, but subtle role in recent string theory which is in no way a theory that contains only strings. Note that this new concept actually includes ingredients with a variety of different geometrical dimensions [13, p. 165]. To sum up: basic ideas of matter have also to incorporate very different kinds of geometrical structures into the diversity of the elementary particles under consideration. Doubtless, this theoretical design of an "elegant universe" [13] corroborates Seelig's basic conception concerning the individual geometrical shape of every elementary particle.

1.4. A new idea for calculating and predicting particle masses

The expounded state of the art encouraged us to tread new paths in dealing with the physics of elementary particles. Our theory is based on some premises believed to be in a close connection with some experiences that belong to the matter of course in contemporary particle physics. Four statements may condense them:

(A) Corresponding to the vindication of the SM each other rational theory is equally admissible, provided the latter is confirmed in great detail by reliable and well-posted experiments.

(B) There exists no mathematical theory to determine the masses of elementary particles within any of the well-known SM concepts. Neither string and superstring theories (cf. [13, pp. 218, 340]), nor a higher dimensionalized quantum field theory achieve this.

(C) Applications of the SM are restricted to low energy processes with particle masses, the values of which do not exceed 175 GeV, i.e. the value of the top quark. The electron-neutrino masses in an order of magnitude 10^{-11} GeV cannot be accounted for by the SM formalism [8]. Additionally, the SM requires certain particle interactions which remain consistent only, if its numerical parameters are fine-tuned — to better than one part in a million billion (10^{-35}) — to eliminate the most fatal quantum effects.

(D) Quark masses cannot be determined without reference to an exclusive computation scheme. For this reason, in the data listings there are often significant differences even between the mass values of a single quark [11].

These four assertions are enough to legitimate a new concept to calculate and predict the masses of all elementary particles in principle. Our idea is based on the following postulates:

(a) There exists a theoretical basis to calculate the masses of all *known* elementary particles. This basis has been worked out by *Wolfgang Seelig* some years ago and published elsewhere [14, pp. 16–26]. His theory reduces the mass of any elementary particle to an experimentally well-validated reference mass along with specific contributions derived to be valid for the particle under consideration.

(b) Seelig's values of elementary particles, so far as they are numerically available, reproduce all known experimental mass values within the accuracy of measurement. Seelig's theory, however, does not require numerical adjustments like the common version of the SM[2] resulting from a theory "that is so delicately constructed that it falls apart if a number on which it depends is

[2] The common SM requires 19 free parameters that can be adjusted to ensure agreement with measurements; it works without the so-called *supersymmetry*. The latter means that there is for each *bosonic* pattern of string vibration a *fermionic* pattern. After incorporating the basic idea of supersymmetry — bosons and fermions occur in pairs — into the framework of point-particle quantum field theory, the SM is relieved "from the delicate task of tuning numerical parameters in the SM to avoid subtle quantum problems" [13, p. 179]. Unfortunately, any explanation is missing why these particle-pairs have not, as yet, been discovered. Doubtless, their experimental confirmation would be a compelling, albeit circumstantial, piece of evidence for strings, the vibrations of which furnish them with properties like masses and charges.

changed in the fifteenth digit after the decimal point" [13, p. 175]. Strikingly, the same is true for the respective mass values which are obtained by a formal *generalization* of Seelig's result for the relation between the reference mass and, in particular, the mass of an electron.

(c) All experiences with the *generalization* aforementioned give rise to elevate *it* to an axiom. This means that there is an algebraic equation — in the following called *semi-empirical mass formula* (SEMF) — which contains four parameters and is postulated to be valid for all elementary particles. The crucial point is that those four parameters are *integers*. Hence, the construction of this SEMF is independent of any restriction like the assigned energy level noted above. Ergo, the new formula admits executing special symmetry transformations and predicting the masses of unknown particles at every energy level.

Though there are some additional items to be considered [15], statements (a)–(c) lead us to a remarkable operational procedure. Each application of the axiomatic SEMF concerns predictions of adequate values of the four parameters for each known or unknown elementary particle via symmetry considerations. Consequently, reality, i.e. experimental evidence, decides, whether the theoretically predicted mass values are true or only virtual. This "proof" agrees with the practice being quite accustomed to dealing with the SM of elementary particle physics. Furthermore, it is "the only reason, we believe in quantum mechanics, because it yields predictions that have been verified to astounding accuracy" [13, p. 88].

2. SOME LAWS OF SRT AS LIMITING CASE OF GIBBS–FALKIAN DYNAMICS

Einstein's *special theory of relativity* (SRT) defines a particle as a special *energy–momentum transport in vacuum*, thus destroying the very distinction between matter and process. In other words, there exists a well-known mathematical relation $E(\boldsymbol{P})$ between energy and linear momentum which today forges the decisive basis for the experimental link to atomic reality.

The SRT is the experimentally best scrutinized and examined theory in physics [16]. The corresponding *Einstein mechanics* can be summarized by the fundamental relation

$$\boldsymbol{P} = \frac{E}{c^2}\,\boldsymbol{v} \tag{2.1}$$

which differs markedly from the basic relation $\boldsymbol{P} = m\boldsymbol{v}$ of the mass-point mechanics [17, p. 33, 18, p. 111].

This so-called *Einstein fundamental relation* (EFR) contains the entire transported energy E which in turn depends on the transport velocity $\boldsymbol{v}$. It should be stressed that the EFR is not at all only valid for relativistic motions. According to Falk's dynamics, equation (2.1) furnishes a constitutive condition which specifies a physical system as a manifestation of some few universal classes of particles. To identify such classes, a definite system will be selected which moves in a (Dirac) vacuum. The latter is defined by the double constraint of zero-point pressure and temperature

$$\mathrm{d}\boldsymbol{r} \to \boldsymbol{0}; \qquad p \to 0; \qquad T \to 0 \quad \text{(vacuum condition)} \tag{2.2}$$

assumed to be valid under the additional constraint that all processes occur without any influence of the position vector $\boldsymbol{r}$. This means that, for instance, the common acceleration experiments with elementary particles are running along a plain orbit of motion.

Both the limiting procedures for pressure and temperature need to be realized independent of their conjugate quantities volume V and entropy S as well as the assigned chemical potential per particle $\mu^{\mathrm{t}}(T, p)$ of a physical system consisting of N particles.

Equation (2.2) allows the assumption of an *energy–momentum transport* $\boldsymbol{P}(E)$ to be identical with a characteristic motion of "particles" (but also see [2, p. 137]).

The typical "local" status of this theory is characterized by the transport of a certain number of mass-associated "particles" without an explicit "field effect" which means that the gravitational potential — the field force vector $\boldsymbol{F}$ — has no influence on the speed of light c. Considering (2.2), the *Gibbs main equation*

$$\mathrm{d}E = \boldsymbol{v}\cdot\mathrm{d}\boldsymbol{P} - \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{r} + T\,\mathrm{d}S - p\,\mathrm{d}V + \mu^{\mathrm{t}}\,\mathrm{d}N \tag{2.3}$$

will shrink to the simple differential relation

$$\mathrm{d}E = \boldsymbol{v}\cdot\mathrm{d}\mathbf{P} \tag{2.4}$$

In combination with the EFR (2.1), the well-known *energy–momentum relation*

$$E^2 = (c\boldsymbol{P})^2 + E_{\#}^2 \tag{2.5}$$

results by integration starting from the value $\boldsymbol{P} = \boldsymbol{0}$ of the momentum.

Note that Einstein's legendary equivalence relation $E_{\#}/N = m_{\#}c^2$ between the zero-point energy $E_{\#}/N$ per particle and its (*inertial* or *rest*) *mass* $m_{\#}$ holds true for $\boldsymbol{P} = \boldsymbol{0}$ [16, p. 22]. From the different features of "motion" and "state of rest" the *kinetic energy* of the E–$\boldsymbol{P}$ transport is defined as follows:

$$E_{\mathrm{kin}} := E - E_{\#} \tag{2.6}$$

All energy–momentum transports through vacuum are denoted as “*particles*” [5, p. 53] and allow to be classified by means of the respective value of the (inertial) mass $m_\#$ each.

By the way, it is well known that equation (2.5) can be derived directly from the required invariance for all “four forces” associated with the kinematics of the special theory of relativity [16, p. 24]. This fact admirably confirms the efficiency of Falk’s dynamics. In addition it is clear that the starting elements (2.1)–(2.4) are valid in general and by no means restricted to applications which would relate exclusively to the special theory of relativity (which they cover as well, to be sure!). Both equations (2.5) and (2.6), along with the EFR (2.1), permit the arrangement of an extraordinarily important universal classification of “particles”. It quantifies the discrete energy structure of atoms which was first demonstrated by the Frank–Hertz experiment:

- *common* transports:

$$E = E_\#(1-\beta)^{-1/2}; \quad \beta := \left(\frac{\boldsymbol{v}}{c}\right)^2$$
$$\text{e.g., electrons:} \quad E_{\text{kin}} = E_\#\left[(1-\beta)^{-1/2} - 1\right] \tag{2.7}$$

- *ultra-relativistic* transports:

$$E_\#^2 \ll (c\boldsymbol{P})^2$$
$$\text{e.g., photons:} \quad E^2 \approx E_{\text{kin}}^2 \approx (c\boldsymbol{P})^2 \tag{2.8}$$

- *Newtonian* transports:

$$\boldsymbol{P}/N = m_\#\boldsymbol{v}; \quad \beta \ll 1$$
$$\text{e.g., atoms:} \quad \frac{E_{\text{kin}}}{N} = \frac{1}{2} m_\#\boldsymbol{v}^2 \tag{2.9}$$

Eliminating the energy E between equations (2.7) and (2.1) an interesting result arises with

$$\frac{\boldsymbol{P}}{N} = m_\#\boldsymbol{v}(1-\beta)^{-1/2} \tag{2.10}$$

which leads to Planck’s relation [19, p. 118] concerning the notion of force $\boldsymbol{F}/N$ per particle in its relation to the time parameter t.

In *table II* below the most significant data of such transport processes are compiled. The precise results stem from molecular relaxation and chemical reactions and their interactions. Conventional units are used. From the group of “*bodies*” for which $m_\# \neq 0$, only the stable particles are considered with an infinite life expectancy. Following Penrose [20, p. 220], one might try to imagine that $m_\#$ would be a good measure of “quantity of matter”. Unfortunately, the rest mass is not algebraically additive: “If a system splits into two, then the original *rest-mass* is not the sum of the resulting two rest-masses. The π^0-meson has a positive rest-mass, while the rest-masses of each of the two resulting photons is zero.”

All four *basic forces* involved in elementary interactions are mentioned: *electromagnetic* (em), *weak* (w), *strong* (s), and *gravity* (g) forces; they are mediated by *gauge* particles.

Representative values have been compiled in *table II* from Bethge and Schröder [2, p. 12].

In accordance with quantum mechanics, there is an important correspondence between the mass value of a particle and the ability to localize it spatially. In other words: particles with zero mass like a photon can in no way be fixed by any space coordinates. For this reason, there is a significant difference between a *free electron* and an *electron bounded* within the electron shell

TABLE II
Particle properties and basic forces.

Family	Particle	$m_\#$ [MeV·c^{-2}]	Charge	Spin [S]	Interaction
Photon		$< 2\cdot10^{-22}$	$< 5\cdot10^{-30}$	1	em
Leptons	Neutrino ν_e	$2.27\cdot10^{-6}$			
	ν_μ	$9.07\cdot10^{-6}$			
	ν_τ	$20.4\cdot10^{-6}$	0	1/2	w, g
	Electron	0.511	$\pm e$	1/2	em, w, g
Baryons	Proton	938.27	$\pm e$	1/2	all
	Neutron	939.57	0	1/2	all ($\neq$ em)
Newtonian	Mass-points	> 0 optional	0	0	all ($\neq$ w, s)

around the nucleus of an atom. As a typical example the notion *cloud of electrons* is often used metaphorically to consolidate this knowledge. Thus, it is impossible to find the electron of the H atom at a well-posed place related to the nucleus, though there is even an orbit fixed by a well-posed radius like the Bohr radius. In contrast to this *nonlocalization*, a free moving electron outside the atomic entity can be localized in principle depending on the local velocity of the respective electron.

This is the place, where it seems necessary to remember that the notion of the mass at rest $m_\#$ considers two quite different aspects:

(1) Within a discrete value spectrum of elementary particles each particle can be identified by means of a definite $m_\#$ value as the corresponding *energy–momentum transport* being subject to one or more of the four basic forces noted above.

(2) The value of $m_\#$ characterizes the quantum-mechanical property of particle *localization*.

The latter is in no way of statistical nature, but of a probabilistic one. This means that there is a quantum-mechanical wave function introduced to quantify the degree of particle localization. The electron is an appropriate example again: whether in free motion or in co-operation with other bound electrons — in each case it is about an electron!

Note that quantum mechanics allows proving that there is a simple relation between the degree of localization of any particle and its mass value. For example, an extreme small mass value belongs to a particle, the position of which is only identifiable within a certain probability with regard to a definite spatial domain. This particular behavior manifests the true essence of matter denoted as the wave–particle duality.

According to de Broglie's wave mechanics, a moving particle is associated with the wavelength λ of a complementary wave. This idea was suggested on the grounds that electromagnetic fields can be treated as particles, the so-called *photons*. One could, therefore, expect material particles to behave in some circumstances like waves. As a consequence, the *complementarity* is quantified by equation (2.7) inclusive its limiting cases (2.8) and (2.9) and valid for the system, on the one hand, and by de Broglie's set of equations

$$\left|\frac{\boldsymbol{P}}{N}\right| = \frac{h}{\lambda}; \qquad \frac{E}{N} = h\nu \tag{2.11}$$

representing the associated *matter wave* of the particle, on the other hand.

3. SEELIG'S THEORY OF ELEMENTARY MASSES

It should be emphasized that the momentum $\boldsymbol{P}$ and energy E refer to the *moving* particle interpreted as an energy–momentum transport. For this reason, the question concerning the rest mass $m_\#$ of a particle requires to consider only the limiting case of equations (2.7)–(2.9) for vanishing momentum, i.e. $\boldsymbol{P} \to \boldsymbol{0}$. Hence, both relationships (2.12) reduce to a single "double equation"

$$\lambda_\# E_\# = hcN \quad \text{or} \tag{3.1a}$$

$$\lambda_\# m_\# = \frac{h}{c} \tag{3.1b}$$

where the subscript # points to the state at rest. To derive this basic expressions the two equivalencies

$$\lambda_\# \nu_\# \equiv c; \qquad \frac{E_\#}{N} \equiv m_\# c^2 \tag{3.2}$$

were applied to the state at rest associating the wavelength λ with the frequency ν for the wave aspect as well as the energy $E_\#$ with the mass $m_\#$ for the particle aspect.

Both the equations (3.1) were first founded as *main principle* of Seelig's new theory [14, pp. 16–26] concerning the *rest energy* and *rest mass*, respectively, of all elementary particles.

Equation (3.1b) allows one to form an interesting relationship between two different particles. Assuming the quantities $\lambda_\#^0$ and $m_\#^0$ of any *reference particle* to be given, the simple formula

$$m_\# = \frac{\lambda_\#^0}{\lambda_\#} m_\#^0 \tag{3.3}$$

results.

Equation (3.3) is remarkable as to the fact that the ratio $\lambda_\#^0/\lambda_\#$ can be expressed by means of additional physical arguments in analogy to a well-known formula derived long ago for the rest mass of an *unattached electron*:

$$m_{\#\mathrm{e}} = 2\alpha^{-2} m_\#^0; \qquad \frac{\lambda_\#^0}{\lambda_{\#\mathrm{e}}} := 2\alpha^{-2} \tag{3.4}$$

The quantity α is commonly named as *fine-structure constant*, the physical property of which is dubious for years. Perhaps the first access is the option to interpret this constant geometrically following the ideas of N. Bohr and A. Sommerfeld. The crucial point to understand equation (3.4) is the fact that the mass $m_\#^0$ belongs to a certain equivalent energy assigned to the so-called *Rydberg constant* R_∞. Seelig [14, pp. 16 f.]

deserves merit for the idea to attribute R_∞ (expressed in energy units) to a definite particle in the sense of de Broglie's concept of wave–particle duality.

Provided that such a selection of the reference particle is associated with an available value of the wavelength $\lambda_\#^0$, then the *dimensionless* parameter α may be attributed to $\lambda_{\#\mathrm{e}}$ by reference to a pair of characteristic length according to

$$\alpha = \frac{2\pi r_\mathrm{e}}{\lambda_{\#\mathrm{e}}} \tag{3.5}$$

The latter equation involves the classical electron radius r_e which is related to the *Hartree energy*

$$E_\mathrm{h} := \frac{Ne^2}{\varepsilon_0 4\pi r_\mathrm{e}} \tag{3.6}$$

where e and ε_0 denote the *elementary charge* and the *permittivity of vacuum*, respectively.

Not long ago, Wolfgang Seelig realized that equation (3.3) along with equation (3.4) can be arranged in such a way that the result is appropriate to represent a universal valid formula to calculate the mass of rest $m_\#$ for all known *elementary particles*. Seelig's rule

$$m_\# = \Lambda^\mathrm{e} 2\alpha^{-2} m_\#^0 \tag{3.7}$$

is based on some sophisticated considerations, where Λ^e means a function of the ratio $\lambda_\#^\mathrm{e}/\lambda_\#$ that stands for complicated geometrical concepts assumed to be representative for the individual shape of the particle in question.

In order to get a theory being intended for success in practice, Seelig's selection of the reference particle, attributed to the Rydberg constant, is equivalent to the preference of a highly precise physical information: the Rydberg constant which is a fundamental physical quantity arising in the series formulae for the spectral lines of the hydrogen atom. Expressed in common energy units (e.g., in eV) the Rydberg constant R_∞ equals 13.6057 eV.

Starting with a so-called Rydberg H atom, the electron of which is put in a highly excited state of energy, the reference particle can be simply established by addition of a small amount of energy to realize a well-defined limiting case of such a Rydberg atom: the electron consists in the state of *ionization*. If this amount of *energy per particle* is given by R_∞, then equation (3.1a) goes over to the rule

$$\lambda_\infty \equiv \lambda_\#^0 = hcR_\infty^{-1} \tag{3.8}$$

and yields, firstly, the corresponding wavelength λ_∞ of the reference particle. Secondly, the required mass value $m_\#^0 = m_\infty$ follows immediately from equation (3.1b). The respective numerical results are given below:

$$\begin{aligned} \lambda_\infty &= 9.11253541 \cdot 10^{-6}\ \mathrm{cm} \\ m_\infty &= 13.6056981(40)\ \mathrm{eV} \end{aligned} \tag{3.9}$$

Considering equations (3.8) along with (3.7), any elementary mass $m_\#$ per particle can be formally related to both the reference values (3.9) according to the instruction

$$m_\# = \Lambda_\infty^\mathrm{e} 2\alpha^{-2} m_\infty; \quad \Lambda_\infty^\mathrm{e} = \Lambda_\infty^\mathrm{e}\left(\frac{\lambda_\#^\mathrm{e}}{\lambda_\infty}; \frac{\lambda_\infty}{\lambda_\#}\right) \tag{3.10}$$

where the electron rest mass $m_{\#\mathrm{e}}$ resulting from equation (3.10) agrees with equation (3.4), provided that (the Rydberg mass) m_∞ is used as reference mass $m_\#^0$ and Seelig's *geometro-dynamical function* $C_\mathrm{T} \equiv 2\Lambda_\infty^\mathrm{e}(\lambda_\#^\mathrm{e}/\lambda_\infty; \lambda_\infty/\lambda_\#)$ fulfills the identity $\Lambda_\infty^\mathrm{e} = 1$.

Seelig made his theoretical concept for establishing the function $C_\mathrm{T} \equiv 2\Lambda_\infty^\mathrm{e}(\lambda_\#^\mathrm{e}/\lambda_\infty; \lambda_\infty/\lambda_\#)$ in view of the particles' *distinguishable spatial shape* assumed to be manifesting its individuality in a steady-state interaction with other particles at definite energy level. Clearly again, the decisive measure is the degree of agreement between the theoretical and experimental results.

By the way, it should be mentioned that Seelig's insistence on a fundamental connection between any elementary particle and its assigned characteristic geometrical structure in space–time reminds us of one basic idea of modern *string theory*, according to which each elementary particle is in no way point like, but instead consists of a tiny one-dimensional *loop*. This so-called *string* constitutes the particle by its special vibrating, oscillating, dancing dynamics within a distinguished spatial structure (cf. [13, p. 14]). In other words: "... extra dimensional geometry determines fundamental physical attributes like particle masses and charges that we observe in the usual three large space dimensions of common experience" [13, p. 206].

Unfortunately, there is no room for Seelig's sophisticated considerations intending to find an individual solution for each particle as well as some general patterns of the function C_T. Nevertheless, it should be noted that Seelig's theory leads to a new explanation of the inverse fine-structure constant α^{-1}. Thus, the *original* form of *Seelig's basic equation* [14, pp. 30–31]

$$m_\# = C_\mathrm{T} \varsigma^{\beta_\mathrm{T}} m_\infty \tag{3.11}$$

is expressed by means of the *Seelig factor* ς:

$$\varsigma = \frac{(3\pi)^3}{2\pi}\left(1 - \frac{1}{3\pi}\right)^{-1/4} \tag{3.12}$$

where $\beta_{\mathrm{T}} \in \{0; 1; 2; 3\}$ and ς equals α^{-1} numerically. At present, the following numerical results are valid:

$$\varsigma = 137.02874; \qquad \alpha^{-1} = 137.03599$$

All facts notwithstanding, it is notable that equation (3.12) becomes part of a group of relationships each published by some of the most prominent physicists of the last century. A. Eddington, M. Born, A. Landé, P.A.M. Dirac, etc. derived formulas for α^{-1} with the declared intention to understand the structural resemblance to the *elementary charge*. The results sometimes exhibit esoteric features rather than physical reasoning (e.g., Dirac's formula $\alpha^{-1} = 1.5\ln(10^{40})$, cf. [14, p. 77]).

To make preparation for a particular extension of Seelig's results to hitherto unknown masses of well-known elementary particles and even of particles which are allowed to be predictable, a simple mathematical generalization of equation (3.11) may be presented:

$$m_{\#} = 2^a \cdot 3^b \cdot \left(k_{\#} \cdot f_{\mathrm{k}}^c\right) \cdot \varsigma^d \cdot m_{\infty} \tag{3.13}$$

where the *dilatation factor* f_{k} is defined by $f_{\mathrm{k}} = 1/(1 - 1/(3\pi))^{1/2}$.

For the purposes of comparison, Seelig's results, calculated by equation (3.11) and exactly mapped by means of equation (3.13), are appropriately compiled for a lot of elementary particles in *table III* (cf. [14, p. 31]).

4. SEMI-EMPIRICAL MASS FORMULA (SEMF)

Seelig's basic equation in its form (3.13) motivated the authors to try its generalization holding for all proved and detected particles in future. Such an approach may be substantiated by two arguments:

(1) The present status of the respective considerations discussed within actual basic theories like those of *superstrings* and extended *Quantum Chromo-Dynamics* does not hold out the prospect of any concrete success in immediate future. There is neither any promising mathematical formulation of the pertinent physical ideas nor some adequate inspiration of the principles suggested to be true for those advanced theories (cf. [13]). The only utopia that is unambiguously discussed concerns the options to realize particle experiments at energies in the order of magnitude of 10^{16} GeV.

(2) Seelig's theory offers an elementary but highly efficient method distinguished by a uniform algebraic representation and intended to calculate all *known* particle masses within the accuracy of measurement. However, there is no reason imposing restriction a priori on the original application of Seelig's method.

As to the first item, the reader should be reminded that in section 1 a concise overview is outlined with regard to contemporary physics in the field of actual elementary particle research. The main problem is that at present no one knows how to deduce, for instance, from the equations of string theory the extra spatial dimensions which constitute the respective rest masses of the elementary particles via specified string vibrations (cf. [13, pp. 217–218]).

Additionally, part 1 of our Internet report [15] concerning the semi-empirical mass formula (SEMF) is devoted to both the motivation and vindication of equation (3.13). The encouraging results lead us to adopt a positivistic position in handling the well-posed problem to generalize equation (3.13).

The crucial point can be focused by means of three aspects:

(1) The decisive advantage of Seelig's theory in comparison to competed theories is its mathematical simplicity distinguished, nonetheless, by a strict recourse to the basic concepts of de Broglie's wave mechanics and Einstein's particle mechanics. Hence, the credibility of the physical inferences might be seriously based on an *aesthetic* reasoning, provided experimentally accessible results of Seelig's theory agree with measurements.

(2) Seelig's theory allows establishing a well-proved empirical foundation — quite in the strict sense how elementary particle physics uses its Standard Model of matter only with reference to success a posteriori.

(3) Seelig's central result seems universally true:

- The set of particle masses at rest consists of a finite number of elements, where each of them is definitely related to a certain reference mass at rest.
- The proportionality between the mass in question and the reference mass is quantified by a product of factors represented by a power function each. Any mass value differs from others only by different values of that product.

Considering all these arguments the subsequent semi-empirical mass formula (SEMF) is easily derived from equation (3.13) after some simple mathematical manipulations. On that occasion the plot-structure of equa-

TABLE III
Mass values of elementary particles adapted from W. Seelig.

No.	Elementary particle	a	b	c	d	$k_\#$	$m_{\#,\text{Seelig}}$ [MeV$\cdot c^{-2}$]	$m_{\#,\text{experimental}}$ [MeV$\cdot c^{-2}$]
1	Electron e	1	0	0	2	1	0.510945	0.51099907
2	Myon μ	0	1	0	3	1	105.021	105.66
3	Tauon τ	3	0	1/4	3	2π	1 784.7	1 784.2
4	Pion π	2	0	0	3	1	140.028	139.56995
5	Proton p	3	0	1	3	π	930.577	938.27
6	u-Quark	3	0	1/2	2	2	4.204	1.5–5
7	d-Quark	1	−1	0	3	π^{-1}	7.43	3–9
8	Meson mass	0	2	−1/2	3	1	306.35	310
9	Nucleon mass	0	1	2	3	π	369.096	363
10	Vector boson $W^{\pm}$	3	−1	0	4	2π	80 374.12	80 330
11	Neutral boson Z^0	3	−1	3	4	2π	90 662	91 187

tion (3.13) was slightly changed for the following reason: Seelig's theory yields identical masses for *proton* and *neutron*, in contrast to experimentally observed mass values. This fact is supported by Heisenberg's assumption: proton and neutron are two different states (*Zustand*) of the same particle. The same is valid for Ξ^*-particles of the baryon family (spin 1/2). Especially remarkable is this fact for Σ^*-baryons with three different stable states.

The *semi-empirical mass formula* (SEMF) comprises the following equations and comments:

$$m_\# = 2^p \cdot 3^q \cdot (3\pi)^v \cdot F^{w+v'} \cdot m_\infty$$
$$F := \left|1 - \frac{1}{3\pi}\right|^{-1/8}$$

the exponents p, q, v, w are *integers*,

$$v' = \begin{cases} 0 \quad \text{for "one-state" cases} \\ \dfrac{1}{v(1+\sqrt{1+v})^2};\ \dfrac{1}{v} \\ \quad \text{for "two-states" cases} \\ -\dfrac{1}{1+\sqrt{1+v}};\ -\dfrac{1}{2+\sqrt{1+v}};\ 0 \\ \quad \text{for "three-states" cases} \end{cases} \tag{4.1}$$

First, the mathematical structure of the new relationship seems plausible with respect to the enormous discrepancies between the numerical values of the rest masses (or rest energies, respectively). At present, mass values are listed running from the scaling level "tiny" (electron-neutrino: $m_\# = 2.2676164 \cdot 10^{-9}$ GeV$\cdot c^{-2}$) up to the level "huge" (cosmology: 10^{16} GeV$\cdot c^{-2}$). Hence, an exponential dependence on the assigned parameters p, q, v, w is evident. Second, the value spectrum of the rest masses is *discrete*. As a consequence, that property cannot be described by a common mathematical function with continuous variables.

The most striking feature of the SEMF concerns the four exponents p, q, v, w suggested to be *integers*. In other words: the individual mass values, each assigned to one of the known or unknown elementary particles, are characterized by a *quadruplet* (p, q, v, w) of integers each. It should be emphasized that every single one of those numbers has no physical meaning. This amazing conclusion is the result of the mathematical structure of the SEMF: it is easy to prove that the direct relationship between the mass value $m_\#$ in question and a given reference mass value m_∞ offers the option to replace m_∞ with another available reference mass m'_∞ without changing $m_\#$. Due to this substitution the corresponding quadruplet (p, q, v, w) of integers turns into the quadruplet (p', q', v', w') of integers again.[3] It is evident

[3] An idea that suggests itself may be the option to determine each quadruplet by common *statistical* methods presupposing that there is a continuous mass spectrum of the elementary particles. However, the latter assumption is seriously in error for the particle masses at rest. It is crucial for elementary particle physics that such masses only exist within a *discrete* mass spectrum. This is in particular true for the Gibbs–Falkian dynamics proved to belong to the fundamentals of the SEMF. All transformation rules of this formula — resulting in a set of sophisticated constraints for the quadruplets of all known elementary particles — depend strictly on the premiss that the rest masses remain unchanged and have fixed values in order to identify the respective particle even under variable physical conditions. The only meaning of the quadruplet numbers concerns these two basic properties: *identity* and *constancy*. In reality, of course, this is an illusion, as F. Nietzsche had perceptively stated as early as the second half of the 19th century (cf. Menschliches, Allzumenschliches, Vol. I, Item 19 — *The number*). Nevertheless, identity and constancy belong to those terms which are at present constitutive for the human understanding of Nature and its scientific description by means of mathematics, i.e. numbers.

TABLE IV
Relevant data defining the characteristic groups of known particles.

Groups	Charge [e-units]	Value range of masses [MeV·c^{-2}]	Spin [spin-units]	Number of particles	Averaged and maximum deviation [%]
Leptons#	-1	0.51094 (e) to 1 784.5 (τ)	1/2	3	0.089; 0.2
Quarks	$-2/3$; 1/3	1.5 (u) to 173 800 (t)	1/2 and 0	6	0§
Gauge bosons	0; ±1	80 330 ($W^{\pm}$) to 91 187 (Z)	1	2	0.01; 0.46*
Baryons	-1; 0; $+1$	938.27 (p) to 2 285 (Λ_C^+)	1/2; 3/2	16	0.0743; 0.45
Mesons	0; ±1	134.98 (π°) to 9 460.4 (Υ)	1 and 0	22	0.1470; 0.67

Without the corresponding neutrinos (see *table V*).

§ Within the accuracy of measurement.

* Considering the *width* of the Z-particle with an averaged value of 2 491 $\pm$ 0.007 MeV [21] the agreement between the best experimental value and the calculated one is complete.

TABLE V
Neutrino masses.

Neutrino	$\nu_e(-1,-1,0,0)$	$\nu_\mu(1,-1,0,0)$	$\nu_\tau(-1,1,0,0)$	$\nu_?(1,1,0,0)$
Neutrino mass [eV·c^{-2}]	2.2676164	9.070465	20.4085472	81.63418884
Mass-ratio of neutrino/electron	$4.4376\cdot10^{-6}$	$17.750\cdot10^{-6}$	$29.9385\cdot10^{-6}$	$159.754\cdot10^{-6}$

that this transformational property may facilitate to track down some unknown symmetries with respect to the wide variety of elementary particles. Furthermore, one cannot rule out the possibility to find a so-called *Auswahlregel* concerning unknown particles and their rest masses. Authors intend to deal with this subject in a subsequent paper. Notwithstanding, it belongs to the objectives of this paper to demonstrate — in particular for m_∞ — that there is a kind of symmetry arising with the quadruplets (p, q, v, w) of integers for all known elementary particles and even allowing us to predict a lot of unknown particles with their masses at rest.

The "multiple states" are considered by the exponent v'. Equation (3.14) results in 938.24 MeV·c^{-2} for the *proton* mass and in 939.62 MeV·c^{-2} for the *neutron* mass. The agreement between empirical and calculated mass values of both elementary particles is excellent. For both states of Ξ^*-particle the calculation shows values of 1 530.88 MeV·c^{-2} and 1 533.12 MeV·c^{-2}, respectively. Considering the experimental uncertainty, the empirical mass difference is 2.92 MeV·c^{-2} unlike the calculated difference of 2.25 MeV·c^{-2}. The masses both remain within 0.1 % of their respective experimental values (0.04 % and 0.08 %). Equation (3.14) gives mass values for the "three states" of Σ^*-particle: 1 382.97, 1 383.88 and 1 387.64 MeV·c^{-2}. The agreement with their empirical values is very amazing (0.01 %, 0.01 %, 0.03 %). In addition, the mass differences within the uncertainty of the measurement appear acceptable.

For comparison the mass values of all known elementary particles were invoked. *Table IV* offers a compilation of all relevant data defining the characteristic groups and their numbers of particles involved.

To work with the SEMF (3.14) the *quadruplet* (p, q, v, w) for each particle needs to be established. This problem can be easily solved via numerical iterations that converge rapidly.

Then, considering 43 stable elementary particles (without neutrinos and quarks), the comparison between the measurements of the rest masses and the pertinent values calculated by the SEMF (3.14) leads to a weighted *averaged deviation of 0.0835 %*. The overall *maximum* error came to *0.67 %*.

The results show first that only the following exponent values occur: $p \in \{-3; -2; -1; 0; 1; 2; 3\}$ and $q \in \{-2; -1; 0; 1; 2\}$. This fact allows us to summarize the empirical results of elementary particle-masses in *table VI* (this table gives the symbols and the exponents v and w, too).

Obviously, it makes sense to assume as a working hypothesis that all elementary particles (the undiscovered ones, too) let be placed in this *p, q value cross*. This is tantamount to the introduction of an *Auswahlregel* noted above.

TABLE VI
SEMF representation of elementary particle masses (the predicted including).

(*q*) (*p*)	-2	-1	0	1	2
-3			Ξ^-(9;20)		Λ(8;11) Σ(8;16) Ξ^0(8;23) Δ(8;18) μ (7;3)
-2			Λ_c(9;10)		
-1	η (9;15) →	Σ^* (9; 3) Ξ^* (9;10) Glueball (9;15) e-neutrino (0;0)	K^+ (8;11) K^0 (8;12)	$\pi^\pm$ (7;2) π^0 (7;0) τ-neutrino (0;0)	e (4; 4)
0	p (9;4) n (9;4)	η_C (9;8) J/Ψ (9;11)	η' (8;9) K^* (8;4) Φ (8;13) Υ (9;12) W (10;5) Z(10;14) R(0;0) Photon(O [-90];0)		
1	Glueball (9;17)	μ-neutrino (0;0) ← Γ (9;17 ?)	Ω (8;–1) D (8;7) D_S (8;11) D^* (8;12) *u-Quark (5;0)* *t-Quark(10;10)*	?-neutrino (0;0) B (8;3) B_S (8;4)	τ (7;7)
2			*d-Quark (5;0)* *b-Quark (8;15)*		
3			*s-Quark (6;0)* *c-Quark (7;31)* ρ (7;5) ω (7;6)		Lepton(4;3?)

For reasons of systematic the reference mass m_∞ may be assigned to an elementary particle denoted as *Rydberg particle* $R(0,0,0,0)$. As to the ratio m_∞/m_{el}, the value $2.6624{\cdot}10^{-5}$ indicates the quantum-mechanically founded nonlocalized behavior of the *bound electron* in the state of ionization.

As to the six quarks, there are some notable symmetries arising in *table VI*. Three pairs of quarks appear in the three fields with $q=0$ below the row $p=0$. Each pair consists of a light quark and a heavy quark, whereby two pairs contain the lightest and the heaviest particle of the two kinds of quarks. The third pair is composed of the two particles with the mean mass values of the two kinds each.

Another sort of symmetry arises with respect to the *light gray* colored area around the central $(p = 0, q = 0)$ area of this *particle cross*.

The left part of this symmetry arrangement leads us to particles which are distinguished without exception by a given exponent $v = 9$ along with all the pair combinations $(p; q = -1, 1; -2, -1)$ and independent of the value of the fourth exponent w.

Concerning SEMF, the present state of the art was confirmed by Vaccario and Weingarten. These US authors recently published mass values of two new elementary particles and denoted them as *glueballs*. ("We evaluate the infinite volume, continuum limit of glueball masses in the valence (quenched) approximation to lattice QCD. For the lightest scalar and tensor states we obtain masses of $1\,648 \pm 58$ MeV$\cdot c^{-2}$ and $2\,267 \pm 104$ MeV$\cdot c^{-2}$, respectively" [22]). Their *glueball mass* informations allow to be reproduced by SEMF within the range of experimental uncertainty.

A first conclusion of this amazing result concerns the very fact that in the pertinent $(-1, -2)$ and $(-1, -1)$ areas the two particle masses $\eta(9; 15)$ and *Glueball* $(9; 15)$ are now proved to exist. Thus, Vaccario and Weingarten's second proved mass *Glueball* $(9; 17)$ leads us to surmise the existence of a particle $\Gamma(9; 17)$ with a mass assigned to the $(1, -1)$ area.

In our web-site [15] we put several particles up for discussion following from some considerations about further symmetry properties. Here, we confine ourselves to only two examples which are, indeed, of general interest.

Unlike the left part of the particle cross, where the exponent v is always fixed by the value $v = 9$, the right part (p, q) areas are distinguished by *different* v values. Obviously, the masses of the three well-known leptons e, μ, and τ are fixed in areas for which the second parameter q is constant $(q = 2)$. The respective quadruplets (p, q, v, w) might indicate an immediate affinity between the μ- and τ-particle compared with the electron. In case that this is true, the sequence of the three leptons around the p-axis along with the special values of their parameters leads us to suppose that there exists a *fourth* lepton marked in the $(3; 2)$ area [23, p. 207]. Its mass value is given by the SEMF to 8.0613 MeV$\cdot c^{-2}$ according to the quadruplet $(3, 2, 4, 3)$ that now evidences a close affinity to the electron.

Recently, the opportunity has arisen to predict also masses of *neutrinos* by means of SEMF within the Second Symmetry hypothesis. Starting from the theoretically as well as empirically based and recently published upper limit for the mass value of the electron-neutrino ν_e (electron-neutrino-mass < 2.5 eV$\cdot c^{-2}$), we find an excellent agreement with the mass of a particle assumed to be identified by the notation $\nu_e(-1, -1, 0, 0)$. Our result $m(\nu_e) = 2.2676$ eV$\cdot c^{-2}$ corresponds to the reference data within the published bounds [9].

Following from the proof of existing neutrino masses at rest, we are interested in the locations of the additional well-known neutrinos ν_μ and ν_τ at corresponding (p, q) areas. All informations available for ν_e, ν_μ, and ν_τ are inconsistent [21]. At present, the cited mass values established in particle physics emerge sometimes as *complex* numbers, even for ν_e. This is certainly erroneous. Hence, we risk to predict values of rest masses serving for purposes of orientation and resulting from the Second Symmetry hypothesis. The latter leads us directly from the SEMF to the two quadruplets $\nu_\mu(1, -1, 0, 0)$, $\nu_\tau(-1, 1, 0, 0)$,[4] and, amazingly, to a *fourth neutrino* $\nu_?(1, 1, 0, 0)$, whose postulated existence follows from the *particle cross* immediately by evidence [23, pp. 72/207].

5. HIGGS PARTICLES

This is, where the question arises, whether the alleged existence of the ominous *Higgs particle* can be substantiated by the SEMF or exposed as a chimera. In modern physics many scientists take the view that there should

[4] "Neutrinos, they are very small. They have no charge and have no mass And do not interact at all. They snub the most exquisite gas, Ignore the most substantial wall, Cold shoulder steel and sounding brass, ..." John Updike's well-known poem (in: From Telephones Poles and other Poems) is now proved to be erroneous: there is "direct evidence that the tau neutrino is one of the building blocks of nature and that it reacts with other particles in accordance with our current scientific theory of particle interactions". This striking result comes from an international collaboration of scientists at the Department of Energy's Fermi National Accelerator Laboratory and was announced on July 21, 2000 (http://www.fnal.gov/directorate/public_affairs/press_releases/donut.html). Four instances were reported on a neutrino interacting with an atomic nucleus to produce a charged particle called a tau lepton (ν_τ) whose main signature is a track with a kink, indicating the decay of the tau lepton shortly after its creation. Although their scattering experiments indicate that tau-neutrinos are unbiasedly suspected of having mass, the Fermilab physicists are not in a position to offer reliable information either of its mass value, or at least of its mass tendency within the neutrino family. Thus, the mass value of ν_τ derived from SEMF and presented in *table V* is the only credible data hitherto known. This is true in particular with reference to the unconvincingly large upper limit of 16 MeV$\cdot c^{-2}$ allegedly founded by particle physics.

be a basic mass alone allowing to distinguish all other particle masses from it by some dimensionless factors [24, p. 164]. If this basic mass could be identified with the value of the respective *Higgs field*, then the mass of any elementary particle should follow from its coupling with this field. For as much as this idea is persuasive with reference to its apparent simplicity along with relevant experimental inferences, things look bad for the case, where the concept of a Higgs field does not correspond with reality.

Note that this theoretical approach follows above all from the urge that quantum theories, as a rule, need to be renormalizable[5] in order to avoid a priori the disastrous infinities mentioned above. Renormalization is guaranteed by the *Glasham–Salam–Weinberg theory.* This (GSW) model concerns different values of the basic forces occurring simultaneously in electromagnetic fields and weak interactions between elementary particles like protons and antiprotons at high-energetic collision processes (cf. [26, p. 194, 26, p. 76]).

This is not the place to enter into details of this sophisticated physical approach. Anyhow, its relevant assumptions and results matter rather more than its other elements. Following the GSW theory, at least one neutral particle with just one spin state ($S \equiv 0$) — the Higgs particle — should occur as a consequence of the *Lagrange density* of the system under consideration combined with the respective elementary Higgs field. The latter is nothing else than a special kind of a scalar background field being capable of yielding some symmetry properties of the Lagrange density with respect to certain parameters of the system. More precisely: this so-called *Higgs mechanism* does even establish a *spontaneously broken* symmetry together with a *local* gauge invariance of the Lagrange density.

The Higgs mechanism is proved to work just under the essential pre-condition that there is *local* symmetry before the system turns into a state of spontaneously broken symmetry. The crucial point: *local* symmetry means the existence of gauge particles like photons or W-particles each with *zero* mass at rest. Forces with *infinite* extension are the consequence. The reverse is also true: forces with *finite* extension only permit *global* symmetry. A clear conclusion of this alternate behavior of matter was drawn from Jeffrey Goldstone in 1961/62.

[5] The term *renormalization* is crucial for quantum mechanics. For this reason R. Feynman's opinion [25, p. 128] is essential: "I suspect that renormalization is not mathematically legitimate. . . . no matter how clever the word, it is what I would call a dippy process."

An extended version of his theorem states that a spontaneously broken gauge theory just remains renormalizable and leads to heavy Higgs particles, provided the system allows to presuppose *local* symmetry along with *massless* gauge particles (cf. [28, p. 402]). Clearly, the GSW model fulfils this precondition for *photons* as zero-mass gauge particles of electromagnetic fields.

Without any doubt, the most important result of the GSW model refers to two actual issues:

(1) The GSW model offers a transparent mathematical method allowing systematic studies on the development of *Grand Unified Theories* (GUTs). At present, the latter serves to unify three of the *four basic forces* of matter involved in elementary particle interactions, i.e. the electromagnetic force, the weak force, and the strong force (cf. [27, pp. 77 f.]).

(2) The GSW model admits of the experimental proof concerning certain scattering processes as, for instance, they happen if *neutrons* interact with *neutrinos* without conversion of the particles involved.

As to item (2), theory and experiment agree well, in case the theoretical results fit the respective measurements by means of a *free physical parameter* Θ — the so-called *Weinberg angle.* This property is a characteristic of the GSW theory. Due to the fact that this parameter cannot be determined by the GSW theory itself, Θ hides the special properties of the Higgs particle appearing at best under the particular circumstances as they can be parameterized by Θ.

For this reason, the excellent agreement between the results of fitted equations and precise measurements indicates in no way that the GSW theory exactly model reality. The situation is similar to the common experience with a chemically reacting gas under definite conditions for temperature and pressure. Its molar equilibrium composition can be precisely calculated by means of the ideal gas equation of state though it is well known that the latter is just a pure mathematical idealization without any serious relationship to real gases. Thus, as to item (1), the crucial point is the diametrical contrast between the physically exact solution of a well-posed problem and its approximation, even if excellent.

Within the mathematical apparatus of the GUTs the Weinberg angle is no longer a theoretically undetermined free parameter. Moreover, a general feature of all GUTs is evident: they mingle the sources of the three basic forces. Thus, there is a relationship between the numbers of leptons as the cause of the electroweak interactions and quarks as the source of the strong ones. As a consequence, two decisive touchstones of the GUTs are theoretically available and, in principle, experimentally accessible:

- *first*, the prognosis that *protons* decay,
- *second*, magnetic *monopoles* exist.

Despite an eminent expenditure of costs and experimental ingenuity during the last 20 years, it is a matter of fact that both predictions could never be confirmed. This is a grave event. The question remains, of course, whether there is an assumption in the mathematics applied, although physically in error. Nowadays, the latter seems increasingly probable. Therefore, all theories derived from the GSW model are subject to Goldstone's theorem. According to it the concept of Higgs particles and, as a consequence, the GUTs immediately depend on our knowledge of the exact value assigned to the rest mass of each photon.

An interesting inference may be immediately drawn from the SEMF in accordance with Seelig's equation (3.13): there is no elementary particle with a *zero* value of its mass at rest. This surprising result contradicts all theoretical and practical considerations still prevailing at present in physics concerning *photons* (and also *gravitons* in case gravitational forces manifest themselves by a "force particle" like a gluon or a weak gauge boson).

To be sure, this very opposite crucially influences every serious cosmology as to the unsolved dark matter problem. But what does *zero* mean in any mathematical approach to reality?[6] Corresponding to the SEMF it is easy to establish the rest mass of a photon by the quadruplet $(0, 0, -10, 0)$ or, say, $(0, 0, \mathrm{O}[-90], 0)$, where $\mathrm{O}[-90]$ denotes *order of magnitude* $[-90]$.[7] As to the very immense number of photons believed to exist in the universe, the latter quadruplet is indeed much more probable than the former one.

Such a conclusion means, however, that in any case there is no photon with *zero* mass at rest. Obviously, for a *single* photon the discrepancy between *zero* mass and $(0, 0, -50, 0)$ mass is irrelevant in practice. On the other hand, any particle system of a huge number of photons distinguished by *nonzero* masses may be responsible for a phenomenon recently discussed under the term *tired light*. This means that along its way through the infinities of the universe light is subject to irreversibility just as well as all matter. For some years this interpretation is supported by Gibbs–Falkian dynamics (cf. [6, chapter 9]). Furthermore, there are some indications that certain new evolutions in modern superstring theory are tending to consider the mass value of a photon rather extremely tiny than exactly zero. Substituting the zero-dimensional point-particle concept of common quantum mechanics by means of the extended one-dimensional strings — the key-term of superstring theories — leads in principle only to a *nonzero* solution of the mass problem. The reason for this far-reaching inference lies in the connection between the specified string vibrations and their assigned particle properties like masses or charges. It is evident that these characteristic vibrations have need — at least in principle — to take into account a minimum of energy or its equivalent mass. This is true for photons, too.

One can say that without going beyond the scope of the present state in physics, our analysis permits no other conclusion: summarizing the key results of the Gibbs–Falkian dynamics, the string theories, and the SEMF along with some crucial experimental experiences, then the existence of Higgs particles is not very likely.

6. CONCLUSIONS

A unified representation of physical systems within their Gibbs–Falkian phase space can be established for all branches of physics. This is even true for sophisticated realizations of so-called *energy–momentum transports* (EMT) manifesting elementary particles in physics. For this case a simple incorporation of Einstein's fundamental relation (EFR) into the specified Gibbs–Falkian dynamics leads to the Einstein mechanics as the basic theory of EMT assumed to be valid under conditions of ultrahigh vacuum. As a result, the function $E(\boldsymbol{P}, E_{\#})$ relates the energy E to the (linear) momentum $\boldsymbol{P}$, where the knowledge of the energy at rest $E_{\#}$ is required in order to calculate a concrete EMT and to distinguish it from the other ones. This is surely equivalent to the specific information about the pertinent value of the rest mass. Thus, the data of this property of matter appears to be equally the keystone of the theoretical edifice in physics. Thrice are the reasons for this:

(i) The resulting $m_{\#}$ values of any well-posed theory of elementary particles are required to agree with the experimental values within the accuracy of measurement.

(ii) The state of matter in question composed of certain elementary particles must fulfill just the physical

[6] In this context C.F. von Weizsäcker even insists that the methods of theoretical physics can only be successful within the frame of their approximations in dealing with the relations between the object in question and its parts (cf. [29, pp. 338 f., 559 f.]).

[7] Compiling the actual data of elementary particles from time to time, the prominent *Particle Data Group* updated for edition 2000 the value $< 2{\cdot}10^{-16}$ eV of the photon mass [11, p. 1]. SEMF with quadruplet $(0, 0, -18, 0)$ equals it.

conditions for the realization of the processes at micro-level, where those particles can exist.

(iii) Item (ii) does in no way exclusively refer to a few number of elementary particles like those producing the lepton–baryon constancy at state conditions, where the most technological processes occur in practice.

A reliable description of reality only succeeds in case the items (i) and (ii) are valid for the whole zoo of elementary particles.

The results presented in this paper confirm these items under state conditions for which the Einstein mechanics is true along with the Gibbs–Falkian dynamics. Additionally, the reliability of physical methods applied, for instance, in thermofluiddynamics can now be attributed to a new and well-posed theory of particle masses at rest. It deserves confidence by its broad applicability to all known elementary particles, but also to unknown ones presumed to exist by some symmetry considerations. Furthermore, the presented considerations are even supported by recent results of superstring theories with respect in particular to the photon mass supposed to be very tiny but unequal zero. This result would prohibit the existence of Higgs particles.

Compared with the usually accepted Standard Model of common particle physics, SEMF generalized from Seelig's theory seems more suitable for extension. This appears to be true especially in view of predicting new particles and their masses as well as of dealing with particles at much larger energy levels than those occurring in daily practice.

Last but not least: the authors admit that they are surprised about the success of the SEMF as for the precision of the calculated particle masses as well as the possibility to draw far-reaching conclusions concerning the mass value of a photon or the existence of Higgs particles. Many signs confirm our results and views. Nevertheless, our ideas of physical reality and its description might not conform to the rules and methods assumed to be conventional in contemporary physics. True, it is possible that our concept and its implementation are simpler than the common ones. They are in any case aimed at making a serious contribution to modern physics whether in quantum theory or in *ironic* science (cf. [30, pp. 7–8] [8]) — ingeniously outlined by Leon Lederman's cryptic puzzle "If the universe is the answer, what is the question?" [31].

[8] "I do not mean to imply that ironic science has no value. Far from it. At its best ironic science, like great art or philosophy or, yes, literary criticism, induces wonder in us; it keeps in awe before the mystery of the universe."

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